DOCUMENT RESUME

ED 085 258 SE 017 090

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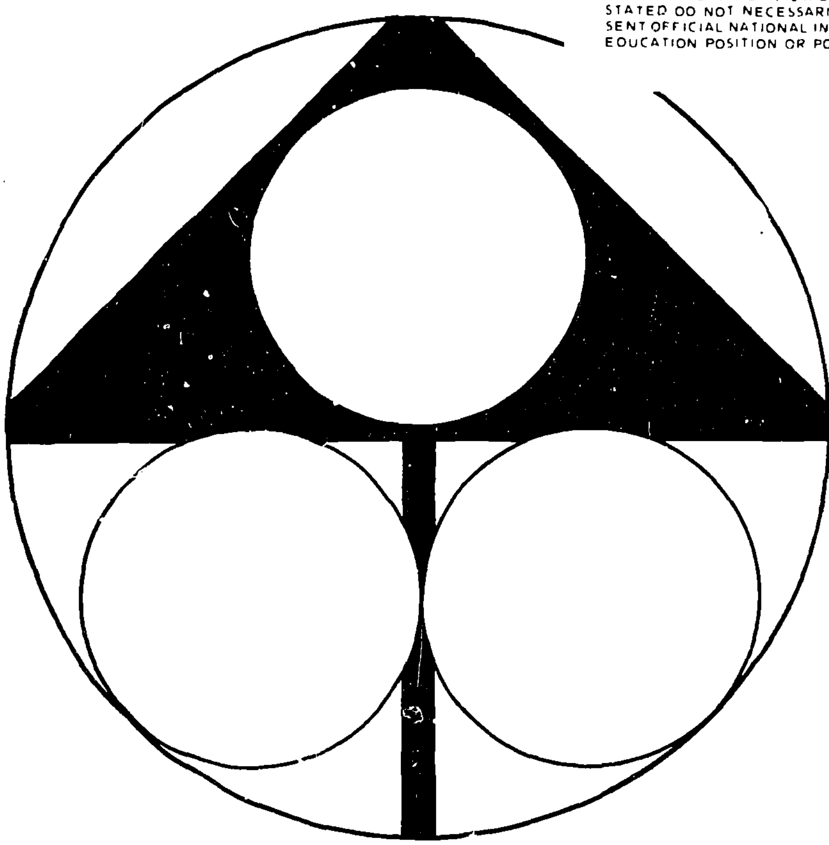
TITLE	Middle School Mathematics Curriculum. A Report of the Orono Conference.
INSTITUTION	Maine Univ., Orono.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	Jul 73
NOTE	105p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	Basic Skills; *Conference Reports; *Curriculum; Curriculum Problems; Elementary School Mathematics; Interdisciplinary Approach; *Intermediate Grades; *Mathematics Education; *Middle Schools; Relevance (Education); Secondary School Mathematics
IDENTIFIERS	National Science Foundation; NSF

ABSTRACT

The focus of this NSF-sponsored conference was on discussion of problems which exist in mathematics education at the middle school level; and to seek some consensus as to future action. Four position papers presented to the conferences, attention to computational skills, relevance in mathematics education to meet societal needs, development of students' mathematical maturity and the establishment of a definite philosophy of mathematics education for this segment of the curriculum are among the topics discussed in these papers. A summary of the ensuing discussions by participants in the conference is also included. (JP)



MIDDLE SCHOOL MATHEMATICS CURRICULUM

A Report of the Orono Conference

Sponsored by
National Science Foundation
University of Maine at Orono

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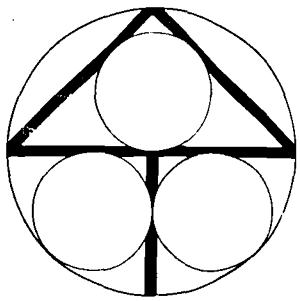

Edited by: Earl M. L. Beard
George S. Cunningham

Sponsored by: National Science Foundation
University of Maine

BACKGROUND OF THE CONFERENCE

The junior high school mathematics of the "pre-sputnik" era was based on the idea that many students would terminate their education at this level. The curriculum was based on an attempt at applying the elementary school material to common applications and was substantially inherited from the 19th century. The only modification to this curriculum during this era was the dropping of applications as they became obsolete. For example: cord wood measurement extracting cube roots and compound interest. The residue was basically a review of what the better student had already learned. The curriculum reform of the '50's and '60's was aimed at this problem along with other curriculum problems and substantially solved the problem by writing materials that introduced new concepts and content. This material was so successful that it was introduced into the elementary school curriculum with the result that junior high school mathematics again became a repetition of prior work.

We at the University of Maine felt that the present junior high school mathematics curriculum failed to meet the primary objective of advancing the student's mathematical maturity on a year by year basis. We felt that it was premature to suppose that our feelings about the present dilemma of the junior high school curriculum were shared by others concerned with educating children. To actually determine whether such a problem existed and if so, to delineate the problem was our purpose in calling the Orono Conference.

The participants to the Orono Conference were drawn from industry, psychology, university mathematics, mathematics education, state and city school systems. They all have shown by their writing and activities an interest in the mathematics curriculum at the precollege level.

ORGANIZATION OF THE CONFERENCE

Invited Participants*

Dr. Earl M. L. Beard
Dr. George Baird
Dr. Lawrence Binder
Prof. Marguerite Brydegaard
Dr. L. Ray Carry
Dr. Arnold Chandler
Dr. John J. Cody
Dr. Arthur Coxford
Prof. George Cunningham
Dr. Lucy Davis
Dr. Donald Dessart
Dr. John Dow
Dr. Preston Hammer
Dr. Frank Hawthorne
Dr. Julius H. Hlavaty
Dr. Donald A. Johnson
Dr. Burt Kaufman
Dr. Jeremy Kilpatrick
Dr. Murray Klamkin
Dr. D. R. Lichtenberg
Dr. H. O. Pollak
Dr. Fernand J. Prevost
Dr. Jack Price
Dr. Thomas A. Romberg

Dr. Joseph Scandura
Dr. George Springer
Dr. Les Steffee
Dr. Henry Van Engen
Dr. Irvin E. Vance
Dr. Herbert Vaughan
Dr. Hassler Whitney
Dr. Lauren Woodby

* There were a number of observers present for the presentation of the invited papers and general discussion.

The participants were all housed and all meetings were held on the Orono Campus of the University of Maine.

CONFERENCE SCHEDULE

MONDAY, July 16

6:30 P.M.	Social hour
7:30 P.M.	Dinner
8:30 P.M.	Opening paper: Fernand Prevost - Review of curriculum change and content to the present

TUESDAY, July 17

7-9 A.M.	Breakfast
9 A.M.	Invited paper: Henry Van Engen - Leading the student to mathemathical maturity in the middle school
10:30 A.M.	General Session
NOON	Lunch
1-3 P.M.	Study group work sessions. Suggestions and reactions to the ideas, problems and solutions presented in the morning paper.
3:30-5 P.M.	General session and discussion
5-6 P.M.	Supper

WEDNESDAY, July 18

7-9 A.M.	Breakfast
9 A.M.	Invited paper: Donovan Johnson - What should be introduced into the Middle School Curriculum

10:30 A.M.	General Session
NOON	Lunch
1-3 P.M.	Study group work sessions
3:30-5 P.M.	General discussion session
6:00 P.M.	Lobster steak picnic

THURSDAY, July 19

7-9 A.M.	Breakfast
9 A.M.	Invited paper: George Baird - How to make a curriculum operational
10:30 A.M.	General Session
NOON	Lunch
1-3 P.M.	Meeting with junior high school teachers
3:30:5 P.M.	General discussion session
5-6 P.M.	Supper

FRIDAY, July 20

7-8 A.M.	Breakfast
8 A.M.	Preston Hammer - Conference Summary

Mathematics in the Middle School
Muddling the Mind, or Minding the Muddle?

Fernard J. Prevost
Supervisor of Mathematics
for the State of New Hampshire

I view my task this evening with some awe: I look out at the audience I'm about to address, think of the accumulated years of contributions that are represented out there and wonder why anyone would consider having me address this group. Only George Cunningham could have talked me into it!

I have a total of 16 years of experience to bring to this conference.

I started teaching as some of you planned the so-called revolution.

If anyone in the audience is 55 or older, and I know a couple of you are, you were already making input into the shaping of mathematics education while I was having my diapers changed, and was getting burped.

And so I feel somewhat like the man who was the only survivor of the Johnstown flood. This gentleman made a reputation as a speaker by lecturing how severe the flood had been and how he had survived the ordeal. Upon his death and arrival at the gates of heaven, he asked to address the heavenly host on his favorite topic. "Sure", said St. Peter "but remember - Noah is in the audience".

I feel like a disciple about to address the master - like Peter turning to Christ and saying "have you ever heard of the beati'udes?"

And yet my assignment is to keynote this conference and the awe nothwithstanding, I feel deeply honored and hope I may take some contribution to the worthwhile purpose of this meeting.

And finally, as a prefatory statement, may I ask your indulgence if some of my thoughts sound like echos. There is hardly a participant at this conference I have not heard, read, or by whom I have not somehow been influenced. I trust that if you hear your echo, you will take heart that someone heard you and not judge me as lacking in originality.

With that, let me proceed.

My assignment was to review the course of curriculum change during the past 15 to 20 years. That indeed could prove to be rather boring for you and frustrating for me. You lived it, you caused it, you continue to influence its course.

I have chosen rather to consider some of the issues facing mathematics education in 1973 from the perspective of the last 15 years, and with words penned over the past 70 years.

Born, as I was, to the profession at the time that the CEEB report and its appendices were being released, nurtured in my early years by SMSG, UICSM, NSF Institutes, GCMP, Ding Dong School, and Captain Kangaroo, taken further along by the Madison Project, Dave Page, Pat Suppes and Minnemast, affected by the work of Professor

Van Engen, by the input of Piaget and Papy, and having very recently seen the CEMREL project, the new Fehr materials, and the new program in elementary mathematics coming out of the IGE work, as well as influenced by Sesame Street and Mr. Rogers, I have of necessity formed some positions regarding what mathematics is in the middle school, what it could have been, and what it might yet be.

And so, my colleagues, with trepidation and shaky knees, I turn to the program of the middle school.

What is a middle school?

The Middle School, as my mathematician knows, is in the middle. The question is "In the middle of what?" In some places, the school is in the middle of a broiling controversy over how schools in the district should be organized. In others, the school is in the middle of a population explosion and serves as a holding area or staging area for the next campaign.

Each of you can probably add another example of a school caught in the middle of some problem, labeled a middle school - and serving in one way or another the needs of youngsters aged 10 to 13, or grades 5 to 8.

Administrators of middle schools live by the following credo:

> "My Goal Should Be to Thoroughly Analyze All Situations, Anticipate All Problems Prior to Their Occurrence, Have the Answers for These Problems, And, When Called Upon, Move Swiftly to Solve Them ...
>
> "However, When you Are Up to Your Ass in Alligators, It's difficult to Remind Yourself That Your Initial Objective Should Have Been to Drain the Swamp!"

And so, in my experience, little real planning has been done regarding an appropriate curriculum in mathematics for these children.

Bob Davis once said that 5th and 6th graders are scientists while 7th and 8th graders are engineers. Yet, if we look at the program for grades 5-8, we find little difference in approach to these very different years.

Two of my colleagues and I had the opportunity to state our views on the curriculum problems we saw for the 70's. This was an exciting assignment because we had to prepare one paper and we represented the disciplines of science, social studies, and mathematics.

After several meetings and hours of deliberations we found ourselves agreeing on many points and that was what made it exciting. In particular, we were of one mind regarding the content of the junior high - or of the upper elementary and junior high school years - the middle school. We saw the role of the elementary school being one of providing some basic skills and the middle school as being involved in the expansion and application of these skills. We stated:

> "We feel that the evolving middle school and the present junior high school curriculum will sharpen these process and content skills and emphasize the peculiar techniques of each discipline in the area of problem solving and critical thinking. These will be the years in which the student learns to apply the skills mastered earlier in an effort to build the broad conceptual schemes or structures underlying the various disciplines."

Now that's nothing very new, Moore made similar proposals in 1902 and other writers penned similar ideas.

But listen to Moore from his address in 1902.

> "The Primary Schools. - Would it not be possible for the children in the grades to be trained in power of observation and experiment and reflection and deduction so that always their mathematics should be directly connected with matters of thoroughly concrete character? The response is immediate that this is being done today in the kindergartens and in the better elementary schools. I understand that serious difficulties arise with children of from nine to twelve years of age, who are no longer contented with the simple, concrete methods of earlier years and who, nevertheless, are unable to appreciate the more abstract methods of the later years. These difficulties, some say, are to be met by allowing the mathematics to enter only implicitly in connection with the other objects of the curriculum. But rather the material and methods of the mathematics should be enriched and vitalized. In particular, the grade teachers must make wiser use of the foundations furnished by the kindergarten. The drawing and the paper folding must lead on directly to systematic study of

intuitional geometry, including the construction of models and the elements of mechanical drawing, with simple exercises in geometrical reasoning. The geometry must be closely connected with the numerical and literal arithmetic. The cross-grooved tables of the kindergarten furnish an especially important type of connection, viz., a conventional graphical depiction of any phenomenon in which one magnitude depends upon another. These tables and the similar cross-section blackboards and paper must enter largely into all the mathematics of the grades. The children are to be taught to represent, according to the usual conventions, various familiar and interesting phenomena and to study the properties of the phenomena in the pictures: to know, for example, what concrete meaning attaches to the fact that a graph curve at a certain point is going down or going up or is horizontal. Thus the problems of percentage - interest, etc. - have their depiction in straight or broken line graphs."
(Presidental address to the American Mathematical Society, 1902.)

I think some of us were trying to do that in the 60's. But I would submit that we didn't quite make it.

It's also interesting to note that Moore's plea for graphs - functions if you will - was echoed by the Russian, Khinchin in 1939. These words have just come to us in this country in the translation of some of Khinchin's works. Khinchin stated his point this way:

> The idea of functions is a central theme in the Junior High and should be explored thoroughly through the notion of functional dependence -

This context and this manner of exploration was most recently echoed in Houston by Morris Kline.

In the 60's we had students associating, commuting and distributing but many couldn't add, subtract, multiply and divide. In the 60's we did a beautiful job of teaching the "empty set" but some kids had a tough time in making change. In the 60's many students learned that 7 x 8 = 8 x 7 but didn't know either was 56. Perhaps we didn't do enough with concrete experiences.

The crusaders of the "new math" were right when they felt students needed to know "why" as well as "how" - for without understanding there can be no creativity. They were right when they felt students needed to know more than just how to calculate and to solve problems. Problems are cast in molds. If a person knows the molds, he can solve problems. It is true that we will always need people who can solve problems of a particular mold. But the demands of the future will continue to be for people who can generate new molds for new problems - problems that we cannot now conceive.

Successful math programs in the coming years must continue to stress the "how" and "why" of everything. If the emphasis now seems to be more on the "how" it is because, traditionally, the public judges a student, and thus his teacher, on how well he can add, subtract, multiply and divide rather than whether or not he knows why these algorithms work.

I submit, my friends, that during the 60's society not only accepted but indeed expected and perhaps even demanded what we were doing in new math. Like the correlation between the Roaring 20's and progressive education, the decade '58-'68 was ripe for new math. Then, with our man on the moon, society's expectations changed, our feet were on the ground again, the common man was the important thing, and a return to basics was called for.

Now, I would hope, we've learned a lesson from this. Some of us, at least, forgot that we have responsibilities to many students and that we must meet many demands in any curriculum we structure. We should read Whitehead more carefully - listen:

> "Any serious fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution. It may be delayed for a generation by vested interests or by the passionate attachment of some leaders of thought to the cycle of ideas within which they received their own mental stimulus at an impressionable age. But the law is inexorable that education to be living and effective must be directed to informing pupils with those ideas, and to creating for them those capacities which will enable them to appreciate the current thought of their epoch.
>
> "There is no such thing as a successful system of education in a vacuum, that is to say, a system which is divorced from immediate contact with the existing intellectual atmosphere. Education which is not modern shares the fate of all organic things which are kept too long.
>
> "But the blessed word 'modern' does not really solve our difficulties. What we mean is, relevant to modern thought, either in the ideas imparted or in the aptitudes produced. Something found out only yesterday may not

really be modern in this sense. It may belong to some by-gone system of thought prevalent in a previous age, or, which is very much more likely, it may be too recondite. When we demand that education should be relevant to modern thought, we are referring to thoughts broadly spread throughout cultivated society."
(Presidental address to the London branch of the Mathematical Association, 1913.)

Our curriculum and our texts must meet the desires of teachers and the needs of students. They must reflect the temper of the times - the demands of the public. They must be mathematically sound and pedagogically practical. They must appeal to the abstract while maintaining contact with the concrete They must be suitable for a class of thirty as well as for individualized instruction. They must be colorful and fun, and must be soberly and seriously mathematical. They must be equally adaptable for the slow-learner and the gifted child. They must be hard-bound and consumable. They must be drill-based, and discovery-oriented. They must satisfy the philosophical constraints of Jean Piaget and of the ultra-conservative. They must be suitable for the open school and the self-contained classroom. They must be contemporary and have lots of historical notes. They must use the block approach and have spiral development.

That's pretty large task.

I see the efforts of this conference being addressed to precisely these issues and others which you will generate. Some of the others, I suspect, will be concerns such as:

1) The role of probability and statistics:

- Man's needs
- Creative problem solvers
- The practicality of the subject
- The interest it generates
- The opportunity to look at it scientifically and also from the engineering point of view

Now, to the extent that we have publications like "Statistics by Example", it seems that we can adopt or adapt some of those to meet the interest of our addressed group of students.

2) The cultural aspects of mathematics as seen in Mathematics: A Human Endeavor, by Jacobs:

I trust you are aware that the bulk of the records purchased in this country are bought by Junior High students. Many of these can serve as motivating forces for a study of the mathematics in music. And last year, I used Don McLean's "Vincent" from the American Pie album to tie together Art, Music and Mathematics. Now, we as leaders must look for more of these connections because that's where the kids are at and we have a responsibility to be there, too. And if we start to do this at grade 7, let us say, dear God, let us have the sense to realize that its success there should not lead us to incorporate it into the program at grades 5 and 6.

In the early 1960's we developed exciting programs for the junior high. We introduced new ways of looking at things and novel ways of looking at old things. I'm especially reminded of the use of other bases to review place value. So what did we do? What's good for junior high should be even better for elementary students. Down the chute!

Pretty soon, the junior high was once again the barren wasteland of the curriculum. No really new topics were introduced except as we tried to bring down some high school subjects. That didn't always work. In New Hampshire, 8th grade algebra courses - suitable for probably the top 15% of the kids - were introduced in small schools as well as large schools. Too often the top 50% were in these classes - and only a few could cut it.

I remember reading - one of you may have said it originally -

> Don't cram it all into one year -
> Save some of the "ah ha's" for next year!

But I've digressed, here are some other concerns I have.

3) Assessment and accountability. We are living in an era when we see more and more legislators demanding some kind of assessment and demanding that the schools be accountable. They aren't sure what they mean. All I know is that they are demanding it, and in state after state people are doing it. And I talked with some of my colleagues who are doing it. And the sad part of it is, they're not sure what they're doing. But they're testing kids. They're testing kids on things which they think the legislature wants assessed and trying to be accountable. We are going to have to live with that accountability and we are going to have to be more careful ourselves.

4) And I am also concerned about the role of the computer and the programable calculator and the advent of the $9.95 electronic calculator. The price is coming down and the machines are getting better all the time. It seems as though somewhere in our deliberations we must concern ourselves with the proper role of these devices. Is it appropriate to introduce it in the

5th and 6th grade? Where do we go from there? Do we introduce the computer at all? Just what are we going to do with that aspect of our schools?

5) I think we have to concern ourselves with the area of laboratory experiences and the vehicle that they provide for integrating the mathematics we teach with the sciences and social sciences that our students are studying. This spring term in one of my courses I had the pleasure of having a very creative young teacher who shared with me, as a result of the term projects which I assigned, some work that he had done in integrating science and mathematics. The students had begun (these were 6th graders) and they had begun by looking at the universe and very carefully studying the distances between planets and all of this thing. And then one day he came in and he said to them, "hey, how big do you suppose the earth would be if we made the sum one yard across?" I haven't gotten to him yet about the fact that it should have been a meter in this metric world. How big would the earth be, and how far away would we have to hold it?

Well, the kids really got caught up in that, and they had some great work and a great show of proportion. And they set up their sun, which was a yard across, and then working from the data that they had previously learned they went about and finally constructed these little, tiny circles which represented some of the other planets. And some had to be stuck on their thumb, because that was the only way they could hold them up there .... And so then he took them outside and he said, "okay, how far away do we have to be," and they paced off that distance. And they got down there, and these little dots on their thumbs and the huge sun up there -- and they're holding that little dot up there, you know -- and they're getting some feeling for just what kind of distances we have in this thing we call our universe. I rather think that we have to do more of that and, you know, those kids were having fun in their learning.

6) The use of the history of mathematics. Although the National Council of Teachers of Mathematics has provided us with help in this area, I see very little sign of its implementation. We also must be concerned with the influence of European

programs. My very good friend, James Smith of Muskegon College, has translated Papy's geometry and has this past year in New Hampshire demonstrated that it can be taught very successfully to students on the high school level. But how much have we done with Matematik Modern, Parts 1 and 2? If nothing else, it strikes me that we should use Papy's graphing, or perhaps I should say graphic techniques.

Today's learners live in a pick-and choose world. It is a four-color, turn-the-dial, change-the channel environment that they have been brought into. The length of their attention spans is a function of how long you can keep them interested, involved, and captivated. They want to be where the action is, and teachers of mathematics and publishers of mathematics materials must provide plenty of action. Not every lesson taught or every page in a book can be a spectacular, but they must be more colorful, more intersting, and more enjoyable - more fun - than any that we have gotten by with heretofore, and the mathematics must still be there.

And so I see the task of this conference to begin to outline curricula - or at least discuss proposals of curricula-that will enable us to mind the muddle while assuring that what we do is not muddling the mind. Indeed, we must design interesting and captivating programs that are fun and that will lead the student to mathematical maturity. And we have the task to do it for all students and the responsibility to do it in a manner consistent with society's expectations - and hopefully not a generation too late.

There is currently much "to do" about Individualized Instruction. I say to you frankly that I have spent this past year laboring under the banner:

"Help stamp out individualized instruction."

Though wedded to its philosophical beliefs, I am appalled by its practices. Though taken by its promise, I am repulsed by the results I see.

Too many times, II has substituted one mode of instruction for another, belying what the term would indicate. Every student working at his own pace through a text or a set of worksheets neglects the student who learns well in groups and from a lecture. I have seen many muddled minds in individualized programs! I have seen a lack of fun.

Totally experiential programs neglect the student who is not particularly adept with manipulations but who has a reasonable chance of understanding oral or written explanations.

An II program should provide for at least all of these learning styles and few do!

And what is most bothersome, is that these programs are so often hailed as panaceas and you and I know damn well they're not.

Chesterton stated:

> A man should always question the

> strongest beliefs of his age,
> for these convictions are invariably
> too strong.

I submit we should heed his words with regards to II in 1973.

A recent Washington inspired movement - and there are different connotations for that word - is Career Education. That label bothers me, too. In fact all labels bother me. Education is career education. Why do we suddenly have to put a label on it? I think we in mathematics have a certain sensitivity to labels, anyway. We could have done quite nicely without the terms revolution, new, and modern. In fact we might have done much better.

Finally, and this is not the beginning of part two, may I suggest that we must ever be aware of the problem of teacher education. The middle school is a new innovation, intended to serve a distinct population, a population with peculiar characteristics and needs. What are we to suggest in this area of teacher education as it applies to middle schools?

And now I am quite relieved. I am not sure that I have minded the muddle or muddled your minds. But I thank you for your attention and would like to close with these remarks.

In 1958 a speaker at the Annual Convention of NCTM said that mathematics can be fun. His talk was extremely well received, applauded, and his words forgotten - nothing really happened.

In 1967 no less than 7 speakers at 5 Regional meetings of the NCTM said methematics should be fun. They, too, were well received and still nothing happened.

Now in 1973 we here at this conference in Orono have the opportunity to vow that mathematics will be fun and set ourselves to the task of making sure something happens - at least in the middle schools.

Fostering Mathematical Maturity in
The Middle School Classroom.

H. Van Engen

Emeritus Professor of Education and Mathematics
University of Wisconsin

The subject under consideration for this Conference is as broad as the whole field of education. It involves philosophical questions such as: Why should our schools be concerned about mathematical maturity? There are also mathematical questions such as: What should be the content of the middle school mathematics program?

Our topic involves also questions about adjusting a program to individual differences and the relative emphasis on skills vs. concepts, that is, how de seated are the objectives of a mathematics program for our schools. There are problems of teacher prepar tion and the form of instructional materials used in the classroom and there are pedagogical questions to be explored. The later, pedagogy, has been ingnored, largely, in some of our more notable efforts of the past two decades to reform the program of the K-12 mathematics program in the schools.

To place some of the questions to be considered in this paper in historical perspective it will be necessary to give a capsule summary of the efforts to change the mathematics program in the schools. The emphasis in this summary will be on the more recent developments since a detailed account of all the committees and comissions that have studied the mathematics program over the past eighty years would not serve our purposes.

A Brief Historical Comment

There has been a number of national groups that have made an effort to change the course of the mathematics program in the schools. The first of these made its recommendations in the last quarter of the nineteenth century. For the most part, the recommendations were concerned largely with the placement and reordering of topics in the various grades as well as eliminating some outmoded topics in the school program.

Two of the most recent and most notable efforts, however, were concerned with more than simple reordering and elimination problems. The Commission on Mathematics of the College Entrance Examination Board and the School Mathematics Study Group (SMSG) were concerned with the whole spirit of the school program. Briefly described, their efforts centered on making use of the structure of mathematics, as conceived by the mathematical world, to develop a program for the schools.

We are all creatures of a climate of opinion--a climate which may, and does, shift from time to time. There are two elements of the climate of opinion that held sway in the 1950's and early 1960's that greatly influenced the type of program that seemed to be acceptable during the mid-century.

The first of these factors was the cold war and the success of the USSR in launching the first orbitable object, Sputnik. While it is true that the then felt need for a large pool of scientists was responsible for the College Entrance Examination Board appointing a Commission on Mathematics (The Commission was appointed before Sputnik I was launched.), Sputnik galvanized the efforts of the nation to be first in science and technology as well as war and peace. This event, Sputnik, caused our nation to spend large sums on the reform of the mathematics and science programs of the schools. The goal of the reformers was to Clean up Euclid and the other high school courses. Not until much later did one hear for the first time, "To Hell with Euclid", and this from a foreign source.

The second element in the climate of opinion that greatly influenced the direction of reform was the utter contempt of the principle reformers for any consideration of methods of instruction. To raise a question of methods or pedagogy in a national group took courage; to do so was to raise a question of academic sanity. Furthermore, in the early days of the National Science Foundation (NSF) grants, a request for support that gave credit for a seminar--note not a methods course--in curriculm was looked upon with much suspicion. One just didn't raise any question about methods of instruction.

The climate for curriculm reform is much different today. In the first place, there is not the intensive feeling for the need for immediate results that existed in the late 1950's. In the second place, even the most ardent "Clean up Euclid" advocate will admit that there are some valid pedagogical questions that need to be considered by all teachers of mathematics. Because of this new "climate," it becomes desirable to discuss some pedagogical questions in the light of our most recent efforts to reform mathematics programs before questions of mathematical content are considered. It may be that methods will influence the content of the curriculum.

Pedagogical Considerations

The problems we are about to consider are a combination of mathematics and pedagogy. They could be called mathematico-pedagogical problems. The first of these problems to consider is the role of a mathematical structure in developing a program for the schools. This, it would seem, is an important consideration in view of the trends of the past two decades.

Structure inevitably involves proof because we can hardly say that a mathematical structure consists of a set of postulates only. The theoreme based on a set of postulates are a part of the structure.

The programs developed in the U. S. have been much too concerned with exhibiting all the basic postualtes of any branch of mathematics under consideration. This drive for mathematical precision is nicely illustrated by a conversation this writer recently had with a mathematician. The position taken was that unless all the basic postulates on which an argument was based are exhibited it should not be called a proof even for the school pupil. On a basis such as this no school pupil could write a poem--at least his product could not be called a poem--because, most certainly, his poem would not meet the standards of an Ogden Nash. This position is absurd. Let me illustrate a proof that would be appropriate for middle school use.

To show that the sum of two odd numbers is an even number, one youngster said, "An odd number must look like this

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. . . .
. . . . .
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The second one could be like this

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. . .
 . .
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Now if we put these two together, we get this

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. . . . . . .
. . . . . . .
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and this is an even number."*

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*I am endebted to Don Lichtenberg of South Florida University for this illustration.

Why shouldn't this be called a proof and why isn't it sufficient for middle school use? Of course any teacher would wnat to supplement this proof with one of the unusual algebraic proofs but insofar as convincing the pupil that the sum of any two odd numbers is an even number, I doubt that the algebraic proof is more convincing. However, we are interested in mathematical maturity. Hence, a second proof may be essential.

A second illustration. To show that the base angles of an isosceles triangle are congruent (equal? or same size) one could argue that an isosceles triangle has one line of symmetry. Furthermore, any figure flipped about a line of symmetry will "fit right on itself" again. Hence, the base angles must be the same size.

Proofs to be proof to the middle school pupil, or any pupil, must have a high degree of acceptability for the pupil. As a kind of counter example to this general principle, taken from the secondary school, let me suggest that proving the theorem that if three points are collinear one must be between the other two is really not a proof for the high school geometry student. He doesn't feel the need for a proof. In this case, the explicit statement of all postulates is a hindrance to developming mathematical maturity and is pedagogically no proof at all.*

* The British have taken a much more relaxed position on the question of axioms and proof than we have in the United States. One need only examine the School Mathematics Project series of texts to see that axioms, explicitly stated, are of little interest to the authors.

Similarly, at the middle school level, the usual proofs for sums and products of the integers are likely to be non-productive. Here the arguments which attempt to relate the algebraic results to the physical world are more convincing for the immature individual.

If the position taken in this paper as regards proof is realistic, and desirable, what then should be the position taken with respect to the basic postulates? My answer is quite simple. In so far as developing structural concepts are concerned, we pay very little attention to the postulates as a list of postulates. This does not mean that the postulates are ignored. In fact, in the middle school the postulates have not been given sufficient attention primarily because the set of mathematical objects under consideration in the program is almost exclusively the set of rational numbers or a subject of the rationals. Middle school pupils should be introduced to sets of objects other than numbers that possess, or do not possess, such properties as commutativity and the existence of inverses. The exclusive attention to numbers in the middle school is not desirable. One sees the importance of such properties as commutativity when it occurs in the study of various sets of objects in a meaningful way. Seeing one dog does not develop a concept of dog. One must see many dogs in varying situations out of which a concept of dog may develop.

Mathematical maturity, whatever it is, is based on a wide and varied experiences with sets of mathematical objects from which common and uncommon characteristics are noted. On the basis of many and varied experiences one can develop general concepts which are at the heart of the subject. Under such conditions, a mathematical operation becomes more than adding and multiplying numbers.* Whatever means are used to develop mathematical maturity, experience and generalization are at the heart of the matter. In many cases modern programs fail to provide for these vital elements. In addition to an over-emphasis on the formal aspects of structure, a recent trend to over-emphasize symbols and terminalogy is evident. For mature individuals an adequate system of symbols may be necessary. However, at the early stages symbolism interfers with idea. As examples of some undesirable practices, I cite present practice relative to the algebra of sets, the insane and undesirable distinction between fractions and fractional numbers, the precision demanded when distinguishing among segments, lines, and their measures as well as other practices too numerous to mention. If the science programs were to follow the lead of their mathematical brothern with respect to precision of terminology, all of us would flunk elementary science because we cannot formulate the difference between a cat and a dog in a manner to satisfy a mammologist. In

*It is worth noting that the addition of 3 x 3 magic squares could serve as a means to broaden the concept of addition since all the properties of addition for the integers are satisfied.

the formative stages of large ideas some ambiguity is essential. The basic rule for every teacher to keep in mind is the fundamental principle of communication; namely, are the communication channels clear of all debris?

Many K-12 programs have difficulty with their own conventions. As examples note those making the fraction-fractional number distinction. Later in the program some refer to multiplying the numerator of a fraction. An obvious impossibility. Also many insist that the pupil distinguish between $\overline{AB}$ as a segment and AB as a measure. A sentence or two later one frequently encounters the term, segment $\overline{AB}$. This is redundency at its worst.

Up to this point, the major concern of this paper has been to caution curriculm developers about being too formal and too precise about matters of no import for the immature student of mathematics. However, one must be constructive and consider the content of the program. In doing this, one must keep in mind that the middle school program does not stand alone. It is built on the elementary school program (grades K-5 inclusive in this instance) and leads into the senior high school program (grades 9-12 inclusive). Problems associated with this linkage must be of vital concern to those attending this conference.

Some "Linkage" Problems Affecting Middle School Programs

It is imperative that some fundamental changes take place in the elementary school program. The elementary schools are too computationally oriented. One reason for the excessive emphasis on computation will be discussed later in this paper. For the present, we accept it as an axiom. There are some realistic observations on computation that must be made at this point. Only after one has worked in the elementary school for a period of time, can one make realistic judgements concerning the great amount of time a teacher necessarily must spend on drill. Furthermore, the only way to releave the elementary school from the routine drill load is to decrease our demands on the number of algorithms for which the first few grades must assume responsibility. This is the only means at our disposal that would provide time for the schools to introduce a significant amount of work on such topics as geometry and probability. The following are two proposals which would lighten the computational load of the K-5 grades and shift some of this burden to the middle school grades (grades 6-8 inclusive).

1. The more complicated parts of the long division algorithm should be shifted to the middle school. Engineers are predicting that within five years some of the pocket computers now selling in the \$60 to \$400 range will sell in the \$5-\$10 range. Why not place the emphasis of the instructional program on the recognition of physical and mathematical situations in which the division of two numbers is called for and let the pocket computer do the work? With such a program, every classroom would need to have a few pocket computers available for pupil use.

Such a program could save at least 20% of the time of the first six grades spent on arithmetic.

2. The responsibility for all the work on multiplication and division of rational numbers should be given to the middle schools.

The reaction of some teachers will be: "It's there now." True! Every junior high school teacher knows what they must reteach how to add two fractions. They why teach it in the first five grades? Furthermore, the more complicated algorithms, if necessary to drill on at all, could be delayed to the last two years of the middle school.

There are other topics that need close examination. However, the two mentioned here give some indication of the kind of thinking about the computational load of the elementary school that must be done. Adoption of a lightened computational load in grades 1-5, would make it possible to make "down to earth" suggestions for topics of a more fundamental nature to be included in the elementary program. Only one possibility will be mentioned here.

The present program is notoriously light in geometry. Any citizen of the U.S. will encounter geometric ideas in daily life situations as often as number ideas are encountered. Yet, the elementary programs, in practice, give too little time to this important subject. The elementary school needs a good geometry program. The problem of how to bring this about will be discussed in some of the following sections of this paper. We turn now to the content questions of the middle school programs.

The Content of the Middle School Mathematics Program

Before trying to characterize the middle school program, it is desirable to make a fundamental distinction that few people make and none of the educational technocrats now so influential in the schools can make.

There is a difference between the Art of Mathematics and the Craft of Mathematics. Only a few of us know the art of mathematics in much the same way as only a few individuals really know and possess the art of poetry. We know very little about how to teach the art but many of us can recognize it. This fact, however, does not relieve the teacher of the responsibility for fostering the art. On the other hand the craft of mathematics can be learned to greater or less degree by all of us. The patrons of our schools demand that the schools teach the craft of mathematics because it has an immediate use in the store and the bank. This does not mean that the people want the mathematics program limited to the craft of mathematics any more than they want the literature program limited to the reading of the newspaper. The craftsmanship aspects of each subject is taken as a lower bound of what the schools must do. In this the schools should not fail.

If we set our sights beyond craftsmanship, we inevitably get into the art of mathematics. Here we must realize our limitations. A story told about Red Grange, famous football player of the twenties is appropo at this time. A newspaper reporter was interviewing Red Grange. The reporter asked Grange why he didn't teach other players to do what he did. To this Grange replied, "Don't give me too much credit for what I can do, because I can't teach other players to do what I can do". Grange beautifully described our dilemma as a teacher. No one knows how to teach the art of mathematics but we are sure of one thing. The art must reside in the ability to do such things as generalize, specialize, guess, solve problems, prove, questioning, and the relation of mathematics to reality. To the extent that we can teach youngsters that these processes are important and to the extent to which we can provide experiences which require the use of these processes, we can teach the art of mathematics. It is in this respect that we can provide a real mathematical spirit for the middle school program.

There is no need to try to go into detail about all the topics that could be used to provide a vehicle for experiences in guessing, etc. There are hundreds of worthwhile topics that could be used. Hence, there is no unique middle school program for fostering mathematical maturity. Many different programs could supply a sound basis for the educational growth of children. We will make a few suggestions, however.

1. Geometry. At the head of the list, we place transformational geometry and vector geometry. Why transformational geometry? Because it provides numerous opportunities to teach the art of mathematics, to supply

useful information, and to bring the pupil in contact with objects and operations which are closed, commutative, non-commutative, inverses, and others.

2. Probability and Statistics. Students should have some experience in how to deal with uncertainty as well as certainty.

3. Basic Language. Most certainly the middle school should continue the pupil's education in the meaning and use of such terms as and, or, not, if----, then----, only if, etc. These ideas are not only useful for maturing mathematical concepts, they are also useful for a citizen who wants to deal intelligently with and more mundane things.

4. Number theory. We choose number theory because it provides so many opportunities to bring the pupil in contact with various aspects of the art of mathematics.

5. Mathematical Applications. Here we wish to provide experiences in which the pupil encounters a mathematician's concept of the relation of mathematics to reality. Not only is this a part of the art of mathematics, it will also supply a vehicle to achieve the objectives of craftsmanship.

6. The Number Program. One often hears that the school program should deal with the real numbers. It is difficult to conceive of the way this could be done. Certainly, it cannot be achieved as a complete ordered field. It would seem that a conceptual mastery of the rational numbers with just a glimpse of the story which

must follow is sufficient for the middle school program.

The above topics are only a few of the many that could be drawn upon as appropriate for inclusion in a middle school program. We turn now to problems associated with any reform movement.

Problems Impeding Reform

In the last five years the schools have been rapidly regressing to a program in mathematics that was in vogue during the twenties and thirties. The reasons for this regression is not hard to find. As was stated earlier, we are all victims of a climate of opinion which exists at any given period of time. During the past few years citizens, boards of education, and legislatures have been reacting to an ever increasing tax burden, a burden mostly attributable to the schools. As a result, schools are under pressure to provide evidence of accomplishment. State legislatures, in some instances, have provided sizable amounts of money to evaluate schools. In other cases, boards of education have employed companies, hastily formed, to show teachers how better results could be achieved. Such terms as contract performance, behavioral objectives, precision teaching, accountability, and many others are children of the efforts to evaluate teaching in the schools. The efforts, to put a measuring stick on the schools have multiplied in spite of the fact, admitted by most competent measurement specialists, that the instruments are at best highly unsatisfactory. From the point of view of mathematics education they are equally unsatisfactory because the instruments in use do measure some of the objectives of a craftsman program but few of the objectives of the art.

The measurement specialist who has emphasized craftsmanship to the exclusion of the artisan aspects of mathematics, has been driving schools back to the mathematics of the 1920's and 1930's. The more recent efforts of the forces of regression usually "sail" under a behavioral objectives banner. Behavioral objectives have a place in evaluating a school program but only if one states in the same breath that they are totally inadequate to measure the more vital aspects of mathematics instruction. Behaviorists should be challenged to write objectives for a course in Shakespeare or for a study of a selected list of poems. Any objective for a class in poetry that is stated in behavioral terms will be insignificant.

The opinion of the lay public also tends to shade any program towards craftsmanship. One sees evidence of this in the numerous attacks on the "new" mathematics. The attack is usually based on the charge that pupils don't learn how to multiply and divide. The charge is true, but maybe children don't learn computational techniques because of a load of monotonous drill and a consequent restriction of the art of mathematics.

The "forces of regression" also include curriculum planners whose zeal to reform outstrips their Knowledge of Mathematics. Nothing can be more detrimental to a sound program in mathematics than to relegate mathematics to the skills section of the curriculum. To do this is to exhibit a very limited knowledge, not only of mathematics, but also of its place in the daily life of the average individual.

The conditions that have held our attention for the past few paragraphs leads one to wonder what could be done to change the direction. To capture the imagination of the school people and the public will not be nearly as easy as it was during the "Sputnik" period. Society today has problems that seem much more in need of the layman's attention than another "go at mathematics". But let's try.

Some Techniques and Instruments for Reform

It is difficult to formulate a program for reform without turning to The Great White Father located in Washington. But the Great Father has many pockets in which he keeps various millions of dollars. Experience has shown that one cannot depend on the U.S. Office of Education for insightful assistance. Figures are not available but the U.S. Office has spent at least 20 million dollars on the elementary school arithmetic program with very detrimental results. So we turn to The National Science Foundation (NSF) and private foundations for help. The record of NSF is not entirely faultless but on the whole it has had a positive effect. So, whatever is proposed here needs financial support.*

Anything that is done in the next decade must have a different tone from that of post efforts. Future work must be able to stand up under the real world of the classroom. Hence, any future efforts must be based on a cooperative effort between school systems and college groups. The systems may be city, county, and state, in fact under the proposal about to be made, there probably should be at least one of each.

I would propose that NSF encourage the establishment of 3-5 centers of curriculm reform. These centers should have most of the following characteristics:

1. Scattered among the 50 states.

2. Purpose: To improve the mathematics program, K-8, in a given system.

3. The target population should vary from center to center. That is, each center would work in one, and only one, of the following: (1) high socio-economic system, (2) low socio-economic system, (3) middle socio-economic system, (4) intercity schools, (5) minority problems such as found in Southwest United States.

4. Each system in which a center is located would agree to provide some support either financial support or personel but most certainly the later.

5. Each system would agree to (a) use the materials produced for a stated period, and (b) cooperate in an inservice teacher program focused on use of new materials.

6. Each center should make an appropriate state supervisor an integral part of the working group for only one purpose, namely, to get an evaluation program established--state wide and system wide.

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*The British seem to produce significant results with little or no government support.

7. The classroom materials produced should be flexible, for example, for each year N and M booklets would be produced. The N booklets would be basic, the teacher to select from M booklets to "fill in" the program.

8. It should be possible to finance each center for ¼ million to ½ million dollars per annum.

9. Research. Provision must be made to support research efforts in mathematics education. There are a few confers were valid and useful research efforts are being made.

If the leaders of each center and the systems were carefully selected, a program such as outlined above could reverse the trend toward a mathematics program characteristics of the thirities attract the attention of the nation to the need for strengthening elementary education programs. The school world needs some reputable group to supply leadership in the form of goals. The schools are being buffeted about by well-meaning groups of various interests and, almost invariably, the individuals have a limited Knowledge of Mathematics and its role in the life of an intellegent citizen. The schools need a Commission on Mathematics (K to 8) to point a direction for the program and to supply a sound philosophical, mathematical, and "child needs" basis for future work in the schools. Let us hope that this conference can supply. the necessary "push" to get such a Commission established.

What Should Be Introduced Intc the Middle School Mathematics Curriculum

Donovan Johnson
University of Minnesota

It seems to me my job is to raise some questions for the conference and to suggest some alternatives that might lay the groundwork for our discussion and for some decisionmaking, decisions about what this conference should propose.

I would like to suggest some of the sources of information that I checked so that you will know that some of the things I say were not pulled out of my hat. I looked at curriculum proposals, the latest one's by SMSG, CSMP, and SSMCIS. I checked two commercial texts, two state department guides and literature in professional publications. In terms of the literature, I found the 69th Yearbook of NSSE (National Society for the Study of Education) on Mathematics Education, and the NCTM Yearbook on the Teaching of Mathematics very fruitfull. One suggestion I made by Romberg in the NCJM Yearbook is that we need good questions, good theory, good models, and good paradigms. Obviously, I can't present all of these, but, I will try to present some questions.

I am making the following assumptions in terms of this conference:

First, that we are primarily concerned with what mathematics every citizen should know, rather than remedial mathematics, mathematics for the gifted or enrichment subjects.

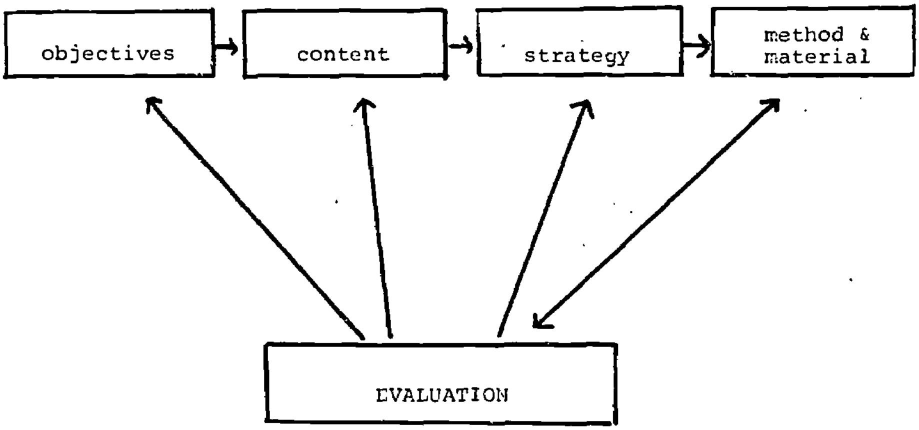
Secondly, we are concerned that the mathematics selected will be sound mathematics, appropriate mathematics and at a level of sophistication and rigor such that learning is rewarding. In fact, previous speakers have mentioned that idea that learning mathematics ought to be a pleasant and fun experience. If one could attain that kind of curriculum, obviously, that would be very helpful to us.

Third, the selection, sequence, and setting for the content may be independent of previous curricula if we are to be creative. I am going to present to you a composite of the topics for the middle school of some of the other projects and the commercial texts as a guide, but I don't think we should be tied to these at all. It would be very helpful if we could be creative and ask ourselves whether there are other topics that are just as appropriate or more appropriate in trying to do the job that we want to have done.

Fourth, our basic curriculum must propose how students are to acquire content, how to optimize the interaction of students and content.

Fifth, the teacher is the crucial factor, who must provide the setting, the questions, the material for exploring, discovering, generalizing and creating mathematical ideas. As someone said yesterday, what happens in the classroom when the door is closed is what is really significant in the mathematical curriculum. No matter what we say should be done or what content we think should be taught, if the teacher cannot put it into effect, cannot get it across to the students, then our work is not fruitful.

Sixth, a means of evaluation needs to be part of the curriculum so that the questions we propose may be explored, tested, and evaluated. And here I would like to refer to Begel and Wilson's article in the Yearbook by the National Society for the Study of Education, where they suggest that the evaluation should be sure to have measures of achievement, retention, attitude and self-concepts. Achievement measured the SMSG evaluation. Even the AAAS first guideline on teacher education curriculums deals with the teacher's self-concept and her acceptance of the students in her classroom. In looking at the curriculum, I would like you to look at this flow chart. It suggests that the curriculum ought to start with objectives or goals. Then we select the topics that will attain those goals and objectives. Next, we develop the mathematical strategy for each topic. A simple illustration is teaching how to find a square root.



You have options as to what mathematical strategy to use in teaching square root. You may want to use the estimation-approximation method or the algorithms where you group in pairs. You may want to use a table of square roots or a slide rule or a computer. After selecting a strategy we next need materials for instruction. Finally there needs to be an evaluation which provides feedback to all aspects of the curriculum.

When we think about curriculum we also ask ourselves, "What are different types of organizations that we might consider?" There are different ways in which we can approach the curriculum development problem.

<u>First</u>: consider your curriculum content to be a

continuous, spiral sequence of mathematical topics arranged in logical order. SMSG is probably a typical example of this.

Second: select the topics on the basis of the needs of every citizen. When this has been done, it has largely been done from the point of view of the slow learner or the general mathematics student. The report in the Mathematics Teacher of November, 1972 in which Edwards, Nichols and Sharp listed the mathematics which they thought every citizen should know is an example of this approach.

Third: a logical, integrated arrangement that emphasizes the structure of mathematics. This integrated arrangement is best illustrated by the CSMP and SSMCIS. The idea of an integrated sequence rather than the system of algebra for a year and geometry for a year seems to me to be a good idea.

Four: a programmed sequence that makes independent, individualized learning possible.

Five: one that we have not explored and for which I cannot really give you an illustration, is an interdisciplinary approach which integrates mathematics with other subjects. Related to this is a proposal for a curriculum which is based on applications.

Sixth: this is a problem-solving approach to the content. In other words, we propose big problems and use them to pull together the mathematical ideas that you need to mathematize the situation. Although this sounds very good in theory, no one has really tried to organize a mathematics curriculum on that basis.

Seventh: An activity learning curriculum in which you have a so-called mathematical laboratory approach.

Now, these approaches are not completely independent. As you can well understand, a comprehensive curriculum, will probably infringe on all of these. At times you may wish to have an interdisciplinary approach. At times you may want to work on the basis of problem-solving approach or activity learning. And so these seven approaches might be considered for any of the topics which are introduced into the curriculum.

A curriculum involved philosophical foundations, psychological processes, pedagogical aspects, and evaluation questions. Let's take a look at some questions under each of these. When it comes to questions under philosophy, obviously we ask questions like these. Why do we teach this topic, this skill, or this process? Is it a prerequisite for further learning of mathematics? Is it used in day-to-day activities? Is it needed to make decisions? Does it entice the student to learn mathematics? Does it really reach the student? Does it provide an opportunity to explore, to create, or to recreate mathematics? Does it build confidence, optimism and a good self-concept?

The objectives which are usually selected for mathematics are often put into categories such as: concepts, problem solving, decision making, learning how to learn attitudes, computation. Note that only concepts and computation are tied to any specific mathematical content. You can probably teach problem-solving, decision making, learning how to learn, in a variety of topics other than those that are in the traditional curriculum. It is computational skill and concepts that are emphasized by behavioral objectives. These are rather narrow objectives and I am not sure we think of them as being the most important ones.

Next lets consider the mathematical aspect of the curriculum. Here are a couple of quotes: CSMP states that -- "substance is more important than style," and "mathematics should be sound in content, enjoyable, and appropriate to their needs and abilities." In "The Mathematical Compentencies and Skills Essential for Englightened Citizens" in the Mathematics Teacher, November, 1972 three categories are established for the mathematical content. First, "select mathematics as a tool for effective citizenship and personal living." The second one is "mathematics as a tool for functioning in the technical and scientific world;" and, third, "mathematics as a system in its own right." So, if you select your content and it can be placed in one of those three categories, you will have to make a judgement as to whether or not all three should be a vital dimension of the mathematics you select.

When we talk about mathematical content we ought to ask questions like these: Is the mathematics correct?; Is it in proper sequence with other topics?; Is it of the proper degree of vigor for the students involved?; Should the same mathematical topics be included for all students with variation in depth and time, to provide for individual differences?

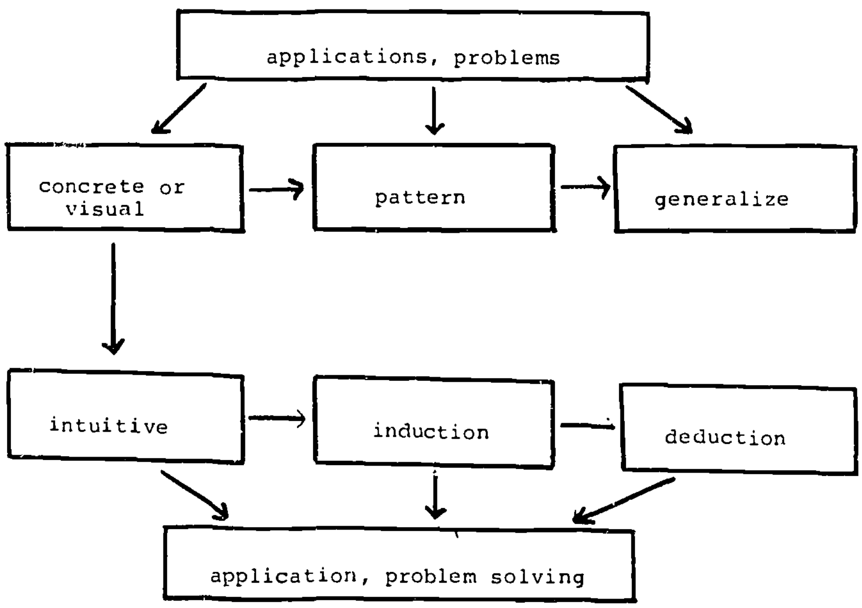
When we consider psychological aspects of the curriculum there are two factors which have been very influential. Bruner's statement of one is, "that ideas can be made comprehensible in some intellectually honest form at any time, at any state of development." Bloom states the second one as "anyone can master ideas and skills, given enough time." When Bruner was interviewed about a year or two ago recognized that he left out the necessity for interest and motivation. So one of the questions we have to ask is "how do we get the student motivated to learn sophisticated ideas?" It is not sufficient, that the mathematics itself be esoteric and of great interest to ourselves. Another major question is "what reaction and participation on the part of the learner is required for learning to take place?" Learning programs with individualized instruction I assume that when students read and answer questions they are reacting and participating and thinking. Mathematic laboratory programs assume that you have to have more or less physical manipulation and participation in order to learn.

Another consideration under psychological aspects is how to instinct students in learning principles that will help them learn mathematics. We do not know what mathematics should be used to learn how to learn mathematics. It seems that we should consciously point out to the students, why we are teaching this the way we are. We should tell them what they have to do in order to learn. Normally, the teacher teaches a lesson from day to day without any explanation to the students about why they need practice, what would be a good way to learn todays lesson or what are some of the difficulties involved in solving problems.

We have assumed an emphasis on structure, will build understanding, improve retention, and make it possible to reconstruct processes. We have very little evidence to indicate the success of this proposal of psychologists. The abstract structure of mathematics, makes it difficult for us to use the structure of mathematics to build retention and understanding.

Under the pedagogical aspects we ought to include group interaction--interaction from teacher to student and from student to student. This may be where individualized instruction really breaks down. Independent activities require a certain level of motivation, a certain level of sophistication, a certain level of reading ability which students may not have. We have been told a great deal about the ineffectiveness of the lecture method but lets not eliminate oral communication. Oral communication may be the best way to communicate ideas -- because you can repeat, you can emphasize, you can use gestures, you can use chalkboards, overhead projector, you can combine oral instruction with many other aspects of communication. So let us not sell short the possibilities of the group instruction if it is properly used.

Another kind of instruction that I wish we could emphasize a little more is student instruction, where students teach their peers. It would be easy, obviously, to exploit students in this method. If you recall your experience when you began teaching, you recognize that one of the best ways to learn something is to teach it. The same thing would be true with respect to our students. The student who is teaching his peers is learning and mastering the subject matter, learning communication skills, learning how to interact with other individuals at the same time that he is mastering mathematics. And maybe other students can listen and understand their peer better than he can the instructor, particularly in a very large class. It has been suggested that we ought to have a so-called systems development for each topic which suggests the optimum materials, and the best sequence for developing that topic. One of the things we can do is provide teachers with materials, ideas, and a sequence like the chart below: then let the teacher make choices to find what works for her.



The sequence is from the concrete or visual to an intuition is the beginning level. If you take the pattern and make an analysis of it, you ought to be able to find some probable answers whether you are looking for a formula, a geometric relationship, or a statement. In terms of a generalization, we want the student to outline a proof or make a deduction, so that he can have confidence that his generalization is true. But a deducted generalization is really not the end. The end is the ability of the students to use his deduction, his induction, or even his intuition for applications and problem-solving. And it is here where one then ought to get the pratice you need to attain mastery. This is probably why the Cambridge report said that all computations ought to be practiced in terms of problem-solving. Thus applications may be used to introduce new ideas as well as to apply a new generalization.

There are several proposals as to what should be in a curriculum, and I have tried to compile the topics in which there is most agreement in commercial texts and experiemntal projects. In general, these are the topics that are proposed although they are presented in different approaches with different emphasis. For example, SSMCIS puts heavy emphasis on group and field construction.

MIDDLE SCHOOL TOPICS

1. Number and numeration
2. Operations and algorithms
3. Rational numbers
4. Ratios, Proportion, Percent
5. Sets, Logic
6. Measurement
7. Statistics
8. Graphs
9. and relations
10. Geometry
11. Real number system
12. Probability
13. Symmetry and Translations
14. Number patterns
15. Calculators and Computer Programs
16. Decision making

There is one other suggestion to consider and that is the possibility of an interdisciplinary approach. Whether or not this is possible and what should be the organization in an interdisciplinary approach, I do not know. But there are lots of people who are saying that this is what we ought to do. In the case of the middle school, it might be much more appropriate than, at the upper levels. I am assuming that the middle school is operating very much like the self-contained classrooms. This would make it relatively easy to operate an interdisciplinary curriculum compared to the typical high school arrangement of periods, departments, topics or subjects.

In an interdisciplinary approach, an easy approach is one that combines mathematics and physical science. Many of the activity materials that are out already do this. If you will look at some of the mathematics laboratory activities you will find that they have experiments in which they collect data on the stretching of rubber bands, the pendulum, the lever, pulleys, inclined planes, friction, or power. In all of these you have collected data that can be analyzed for deriving generalizations. It is hard to get at the deductive aspects through these experiments but you can use induction and arrive at generalizations which involve graphs, functions, computing skills, measurements, vectors and statistics.

Similarly in social science there are opportunities for sampling, probability, statistics, formulas, graphs, predictions, measures, maping, and decision making.

A third field might be economics. Topics such as investments, banking, taxes, and insurance might provide the setting for mathematics such as statistics, probability, or even game theory.

The biological sciences is a fourth field for an interdisciplinary approach.

Certainly a great deal of the biological sciences involve sequences, measurements, symmetries, permutations, and correlations.

Other areas of the curriculum have a variety of mathematical ideas that are embedded in these fields. For example, industrial arts and crafts includes formulas, proportions, costs, geometric, configurations, measurements, congruence, and similarity. We might even list physical education as a possibility. In physical education, topics such as proportions, probability, indirect measurement, similarity, and the Phthagorem theorem are used in performance records, costs and sports equipment. In art and music, mathematical topics include curves, transformation, symmetries, oscillations, sine waves, and proportions. A final field for an interdisciplinary approach is the computers. The computer is a major force changing society. Social studies could teach the role of the computer in society and how the future will probably be cha[illegible]d with the use of the computer. The English d[illegible]ent might teach the programming language because [illegible] essentially a way of organizing communication. The Mathematics department could use the computer as a way to solve problems, to organize solutions, to take apart algorithms, in other words, to learn mathematical ideas.

What about alternate topics? One topic might be called variation which would include growth, decay, and change. For example, the pendulum experiment is a good example of variation because you can change the conditions and ask yourself which of these conditions really determine the period of the pendulum. You can increase the weight of the pendulum and maintain its length and nothing changes. You can increase the arc of the length of swing of the pendulum and still it does not change the time for one swing. The only one factor that causes a change in the time of one swing is a change in the length of the pendulum.

A second topics might be game theory even though this is sophisticated mathematics. The possibility of considering strategies and the possibility of making game analysis might be one approach. Other topics might include creation and research. Look at the problem of creation and research and think about it in terms of a way in which mathematics approaches the situation and tries to create new ideas for its students. To get the opportunity for creativity, we ought to have students create mathematics or recreate mathematics. Curves in space might be another way in which we introduce some sophisticated curves that normally are not treated in the middle school. A topic like equivalence obviously covers many of the traditional topics.

HOW TO MAKE A CURRICULUM OPERATIONAL

George Baird
President's Educational Research Council of America

I will start off by saying that many times in real research there is no such think as a mistake. You just simply find out something that does not work, and that's a useful piece of knowledge.

My topic -- how to make curriculum operate. I take that to mean what one does to create quality education programs and achieve with it rapid use by millions of students on a national basis. The mission of the Educational Research Council of America, of which I am a part, is to improve education continuously so that every child can realize his own inherent worth and be able to contribute humanistically, socially, and functionally to the betterment of mankind.

Earlier in this conference we heard that the classroom teacher has to be the writer of the curriculum. My experience leads me to believe that classroom teachers are not in a position to really write broad curricula. The concept of participation is a very good one, but I think it very unfair to ask people to participate in something in which they're not capable. Many of you have been teachers, as I have, and you know that if you teach all day you don't have time on a left-handed, mimeograph, backroom basis to also build a curriculum. In addition, the hard fact is we haven't trained classroom teachers to be curriculum developers, and if we did, they might not be good teachers. What can a classroom teacher do if we believe in this concept of participation? Do what the teacher does well -- find out from the people that develop curriculum -- Does this work? How does it work? With what kind of children does it work? How is it best communicated? How is it best understood? Where are the hangups that the kids really don't understand? It may be logical and sequential and natural to a research mathematician and just a big fubb to a little 3rd grader. A teacher can give the feedback to the curriculum people who develop content, programs, methods, techniques, and can tell how it works, why it works, and why it doesn't work.

It was said during the course of the conference that we need a different curriculum for every community. I don't believe that. I think we need an American education, and I think we need to have a commonality of knowledge. With the mobility of our population, it is very bad for children to move from one great state to another great state and to find that they've never been exposed to decimals or to the metric system. And I think that ignorance is cummulative and if pupils move from system to system long enough and they will have bigger and bigger blocks of ignorance and, ultimately they should be very bad voters, very bad citizens, and very unhappy people. When I was in Shaker Heights, Ohio, we said very clearly that our community was like no other community and that our children were like no other children, and that we were going to build a curriculum unique to Shaker Heights. I really believe that when I was there. I now can't find a thing that Shaker Heights was really unique in the field of education with the children that we had. We had smart kids, we had dull kids, we had apt and interested and we had uninterested -- we had everything that you have in Wyoming or Mississippi. And the kids, in order to earn a living, need the same knowledge. There we were very sure that we were building a curriculum unique to Shaker Heights, Ohio. Those are old preconceptions. I think they are no longer reasonable in the light of reality. I commented about the national mobility.

Some people have even carried the concept of different curriculums even further -- that we are supposed to have individual curriculum for every child. I will say, that this is unreasonable because it is not possible for any teacher to take any thirty children and move them in a totally individual program in any subject, much less in four or five subjects. No teacher has that much human substance to give.

Two additional reasons why schools shouldn't build their own curriculum -- Money and Leadership. Too often, our school leadership, in the larger communities is based on political expedience. And I'm not putting down the superintendents; I'm saying that's the way you have to make the school systems go in a political system. The second thing is that we, as Americans, haven't yet awakened to the fact that education and educational research is going to be more and more expensive. It's expensive now -- far beyond what most people think -- and that until we are indeed willing to invest money to improve the end product of our schools, we're not going to get a vastly different end product from our schools.

One of the speakers hit drill pretty hard. On this subject, I require preciseness of language. If its mindless repetition, I'm against it. If, on the other hand, it's the wise settling of a skill or technique, of that is going to be valuable to the youngster for the rest of his life, then I am for it. Parents want drill. I think further that they're going to get it because, whether we like it or not, there is something to the old saw about "he who pays the fiddler calls the tune." I think that we'd be wise to act more like parents in this particular topic and not only demand drill where it can be helpful but to find out where it can be helpful and what precise situations in which it ought to be helpful.

What is generally regarded by the public as a need for more drill is, in fact, a noneducators way of expressing a wish for greater competencies in math or any other subject.

I've not forgotten my topic on how to make a curriculum operational, but would like to comment on some other things that have been said in discussion at this conference.

Evaluation. Having spent fifteen years with an organization that has a test and evaluation department headed by a genius, in fact, so damned much of a genius that the analysis is explained in such a way that we've never been able to get it down to the language of superintendents. Somehow or other, in our evaluation of curriculum today, we've been so busy writing doctoral dissertations that we haven't been able to communicate to superintendents in a way that they, in turn, could communicate to parents that there was an improvement in the curriculum-even where there was.

Dr. Van Engen mentioned hard bound books as being one of the economically motivated demands of publishers. Well, he's right. They are. They are demanded on the basis of state adoptions. This is another case where the laws of our land have been disastrous to the good of children. We have soft-bound, not necessarily consumable, format that do not easily destroy so they can last as long as hard-bound. But one of the things that happens is in a big hard-bound book -- is that the pupils get 2/3 of the way through the book but they passed in reading, art, fingerpainting, and whatever else they were taking, and so they have to go to the next grade. They go to the next grade having only this little bit of mathematics that they didn't get; but they skip that and go on to the next big book and start at the beginning and this time have a little bit bigger section that they did not get through. But they passed all those other subjects that they're already pretty good at and pretty soon we've fixed them -- this small part has grown so large that it covers a whole year. They'll never study mathematics again. And we've insured it.

We were cautioned not to ignore the child. Amen to that. I think we need only one good reason and that's basically what our whole busines is -- to help the child to be a better person and to make a better world. We could given another one, as always, and that is -- you can't build a curriculum if you really ignore the child, because without his feedback and without some analysis of what's happening, you know very little.

Now, back to the topic -- how are you going to make the curriculum operational? I've got three things to do to make the curriculum operational. I can say it a lot faster than I can do it

(1) you've got to know what you want to do and what to accept as success;

(2) you've got to build a vehicle to do it; and

(3) you have got to go to work and check and keep checking and working until you complete whatever it is you will accept as success. There it is.

If I were to build an institution to make educational curriculum operational now, I would do several things. First, I would not use a college or university as the organization, because there is no one college that has the wisdom in all of the subject fields within the field of mathematics, much less across the board in other subjects. So when you inherit the three great men in your mathematics department, you and the other two, you also inherit the other ones who aren't worth a blast. We can't afford, as a society, to have those incompetents (maybe I shouldn't say that) building the curriculum that our children will study.

Second, I would not use a government-created agency either because, as a tax-paying citizen it seems to me that more and more we are hiring people who, in effect, are living on a specialized system of welfare, where the job description calls for no production, and by simple meeting the job description they keep their jobs.

I would create an institution that would try to depict the advantages of all the organizations that we're ever seen. If possible, I would use techniques from other sectors of our society other than education. Maybe if we take the attitude that we're too poor to afford cheap things, including people, then we'd get a better product.

I don't think I would have any tenure.

I don't think I'd have any guaranteed annual wage.

I would use money as a reward. That is I would pay enough to be able to say "if they cannot produce, I can't afford it."

I would have job descriptions. I think it helps a man to know what he is trying to do. I would also have performance review.

One thing that such an institution would have to have is hard-headed leadership to a mission, and that everybody in the institution knows what the mission is, and can agree on it, and try to do it. If you really want to make a difference in some given direction, then you ought to have the whole team going down the same field at the same time in a co-ordinated effort.

I would not give away the control of the copyright to anything that was produced. I think copyrights are given for a very simple practical reason -- that which belongs to everybody is nobody's. No one can affort to make a market in material that belongs to the common domain because if they invest capital to make a market and somebody else comes in and takes it away from them, they have lost their money, and we do live in a dollar kind of a society. So, I would keep the copyrights to the material and sell the copyright on an exclusive so that somebody can afford to make a market. This has worked for us. We have got about 13 million students using something every day that has been developed at the Educational Research Council. That's not near as big an impact as I would wish. That's pretty substantial -- that's about one out of every five kids in the elementary schools. You know what, we haven't given anyone a penny. In fact, it works the other way. They pay to use it. That is a much nicer situation.

Yesterday our speaker gave a number of groups that he thought would be necessary in an organization that would develop curriculum. I agree with him. If I were building a new institution, I would have scholars -- biased on content. I wouldn't let mathematicians say where it was going to be taught, or how it was going to be taught. I would have people who are specialists in the humanities, people who are specialists in communications, people who are specialists in child learning theory, work on where it ought to be taught, and how it ought to be taught.

I'd have a second group which I call Practitioners. Again, I'd have them advise on problems of implementation and on problems of distribution, facets of management of materials.

I'd have superintendents and curriculum directors.

I'd have teacher-trainers, principals, classroom teachers. I'd make a big effort to get a lot of input from classroom teachers, because they have a goldmine of knowledge, that is mostly overlooked. Many times a classroom teacher is able to do something in her classroom that is really effective and could be used all over the nation, by classroom teachers, if they could know about it. Many times, a very old in age classroom teacher uses a most sparkling, bright new idea. He may have been using it for 45 years and nobody has ever looked at it before. So we shouldn't overlook the power that we have been overlooking of classroom teachers.

I would have a lay group of some kind involved in the organization. I would have statesmen, business and industrial leaders, community leader types in this group. I would not ask them "how do you teach, what do you teach, or where do you teach?" I would ask them "what is it that you want the American kid to be?" "What is the dream that we want for our children?" I think that we would find out from them that, basically, they want their children to have a terrific education and that they don't want Black education or an Indian education or Chicano education or white education -- they don't want those kinds of things. What they really want is a balanced whole education and they will give us encouragement enough to have the courage of our convictions to go ahead and talk about controversial subjects.

I would have a development staff that was a mixture of all these kinds of people inside the field of education -- the scholars of mathematics, of child learning theory field writers, and so on. I'd have demonstration teachers that could go into the classrooms. I'd require the writers of curriculum to be classroom teachers or in the classroom often enough that they really know what the classroom is about. I think one of the things that all of us are prone to do as we get away from our own past, is to talk about with absolute security what went on when we were there.

I'd have a management group that was dedicated to a one tract purpose. In other words, I'd create an institution that knows exactly what it's trying to do -- and which is going to be measured on that basis. I would insist the management be able to hire and to fire.

One of the things I think we are beginning to find out is that education is not good education unless it is balanced education. Knowing the difference between an impulse and a decision is very important. I suspect it could be applied by those of you who are much wiser in mathematics than I am -- even to the actual functioning in mathematics.

One of the things that we've run into in the field of building and implementing curriculum has been resistance. Here is a short list of the kind of resistance encountered: apathy, envy, laziness, institutional inertia, defensiveness of their own kingdom and defensiveness of the fact that they are as they are, and they don't want people to know it. One way that you can get around these resistances is to listen to all of them, pay attention to all of them, and try to deal with every one of them. Because there are so many people who are against any kind of progress at any time for any reason, I think the wisest thing you can do is to just go around resistance. If you can, treat it with a kind of a loving disregard. If you can't crush it. I don't think there is a middle ground. If you really want to have a mission and you want to complete it, I think it's important that you refuse to go away. Don't give up when you think you're right, stay with it; and if you are, ultimately people will come to you.

I'd like to say something about the process of curriculum development; I think the process has to be a kind of cyclical one. Teachers and administration can recognize curriculum that has not been taught, tested, evaluated and then retaught, retested and reevaluated. I think that we have to test curriculum much better than we did in the past. A minimum of 10,000 students for a year on any topic should be about right to get you the first beginning feedback. Ultimately, before it's released to the nation, I would say reevaluate at least one hundred thousand a year or two or three or four or five years, or until you begin to get a diminishing return on your investment. I think that as the material goes into the classroom a revision operation that follows the actual development of the material and its teaching effectiveness should immediately begin. That way one can get immediate feedback from the teacher. Consequently, from that you can go to an in-service teacher training program which is vital to the implementation of new material. Many times we can take a great idea -- I've seen this happen in mathematics -- you take good mathematical content and you put it into the hands of the classroom teacher; she takes it out and teaches it and when you look at it you say, "That's wonderful for teaching but it is no longer mathematics" and it's important that you have that feedback at all times -- that we keep the integrity of the topic that we're trying to do. So I think many things have to be done eight, ten, twelve times.

Now, to make any curriculum meaningful and operational we need a nice combination of drivers and dreamers.

Of people who know they can change the world, and of those who self-concept is one of mature, visionary optimism, faith, enthusiasm, and dedication. I believe that many of us have that kind of person inside of us. I bel eve that when we as a society, not only in mathematics but all subject metter fields, build improved education continuously so that every person can realize his own inherent worth and be able to contribute humanistically, socially and functionally to the betterment of mankind, then and only then can we call ourselves educators.

Conference Summary

Preston C. Hammer
Professor of Computer Science
Pennsylvania State University

This conference, on the mathematical programs for the middle schools, is most timely. Looking back over the years since Sputnik started frenzied activity in U.S.A. science and mathematics education, we can marvel that so much was accomplished as has been and we can understand why it was inevitable that criticisms of the new mathematics should now mount as they have. Gone, today, is the public fervor for reform, shrinking is the federal support of curriculum reformers; the U.S.S.R. is no longer regarded as a serious competitor in space, the U.S.A. finds itself with an oversupply of scientists and engineers. How then can this conference be timely? It is timely because it is <u>not</u> in response to a widely announced national emergency. If we must have major catalysms to rouse us to action on the educational front then we are not perceiving the needs. Education is always of importance, it should never be merely allowed to drift.

Now evidently, Professors Earl Beard and George Cunningham, our hosts, saw that the problems raised by changes in direction of the middle school programs were worthy of attention. They have, accordingly, organized this conference to establish the existence of problems and to seek some consensus as to future action.

Dr. Fernand Prevost fist gave a keynote address in which he presented some of the background of mathematical curriculum reforms in the past 15 years and indicated where some difficulties with the middle school curricula now appear to be.

Then Professor Henry Van Engen gave a lecture on bringing the middle school mathematics pupil to a stage of maturity. Since his lecture is being reported, I merely mention a few highlights, with comments. There are two distinct aspects of mathematics, according to Prof. Van Engen the craftlike and the artistic. The craftmanship is what he feels is now tested and which indicates failure in some tests of new mathematics. Now, the teacher of grades below 5, according to his idea, is overburdened with drill work and should be relieved by placing more drill work in the middle school. Then, the artistic side of mathematics may receive more emphasis in the earlier grades. Specifically Professor Van Engen proposed delaying study of long division to the 6th grade.

Another type of difficulty in elementary mathematics generally, according to Professor Van Engen is the undue amount of fussiness about rigor and niceties of language. Now I agree with Prof. Van Engen that belaboring rigor or expounding on language niceties will usually be useless.

However, I believe greater emphasis should be on using language carefully and in making statements which are correct, in so far as is now known. For example, it is not a good idea to use "associative axioms" when "associativity axioms" is meant. Whether or not a symbol should be called a number or a numeral is not a straightforward question since in general usage we do not draw the distinction and in mathematics instruction the difference between an "object" and a symbol for it is often ignored.

Rigor is often associated with axiom systems. Axiom systems have been very much overrated in mathematics. Professor Van Engen, for example, says that pupils, to achieve maturity, should learn to observe their mathematical environment. The Greeks from Euclid's time on had an axiom system for geometry. The famous parallel postulate was inserted late by Euclid in hopes that it might be proved from other axioms. Now why and how did the Greeks, who knew well and almost venerated the sphere, not see that spherical geometry was non-Euclidean? Why did it take over 2,000 years for this fact to be observed? We destroy the capacity of pupils to observe by presenting "complete" axiom packages to them. The above example shows a colossal failure to observe or to "be mature." The same kind of effects are achieved today by treatment of axiom systems which falsely suggest that the concepts so defined are somehow basically better than others.

A few other comments may be added which arose in discussion of Prof. Van Engen's lecture. Professor Van Engen had suggested that assessment of curricula also be considered.

The problems of interdisciplinary curricula, the "survival" problems of teachers in certain schools, the problems of selecting approaches to instruction, and of teacher support and training were all raised. The importance of research, and the need to help pupils now beyond the reach of the current new mathematics were also put forth.

There is, by far, too much emphasis on the value of mathematics and its power. Mathematics teaching, I claim, should be more realistic and occationally it should be pointed out where mathematics is futile and fails to apply. For example, mathematics is useless in describing most biological and geological shapes. Every day the average person makes many decisions in which mathematics would be of no help. Some little fraction of the time in school should be spent explaining where mathematics fails to apply. The reasons are twofold. First, it is not true that mathematics applies very broadly. Secondly, it is only the shortcomings of mathematics which suggests the need for future development and research.

In his lecture Professor Donovan Johnson suggested many topics which might be included in the curriculum in grades 5-8 and also presented objectives and a flow chart to show how curricula could be developed. Among objectives he listed was the learning of mathematics to meet social needs. It is, in my opinion, unfortunate that conversations about mathematics do not include the non-numerical applications about which many speak. For example, every child of age 10 has some idea of the blood kinship relation. This can be used to illustrate a general non-numerical kind of distance. Frechet, early in this century wrote down a set of axioms for a distance or metric function. In his terms a metric function is non-negative, real-valued, symmetric, and it satisfies the triangular inequality.

This was a useful definition but it also inhibited others from noting that many, if not most distances of social concern, do not satisfy the Frechet conditions. For example, if we measure the distance between two locations by the cost of traveling from and to the other, we have a distance which can be negative-valued, asymmetric, and which violates the triangular inequality.

The kinship distance suggested above is even worse. It is non real-valued, but is indicated by pairs such as (father, son), (father, daughter), (daughter, mother) and so on. It is not symmetric, and a pair of identical twins are zero distance apart genetically but are distinct people. The triangular inequality may be interpreted as a weak transitivity of closeness. That is, if A is close to B and B is close to C, then A is not too far from C. This does not hold for the kinship distance, since two people may share a cousin without being related.

Should such examples be used? It would seem to depend on whether or not a proper background can be laid. The point I was making about Frechet's axioms, if, of course, nothing to worry the pupil about. The discussion of distances between relatives, however, is one way of helping the pupil get a better understanding of a socially important distance.

The lapse of time it takes to go from one place to another is a measure of distances from the one to the other. This also should be easily grasped by pupils. In fact, the general idea that distances measure separation, inaccessibility, or remoteness can be gotten across in a variety of ways and, if not overly formalized, the experience could be useful to the pupil in studying geometrical distances.

Another objective of mathematics education listed by Professor Johnson is learning to learn mathematics. Now this second order approach should be very profitable provided valid ideas are known. I think one major item here is that the pupil should not do precisely only what the teacher recommends. He or she should do some different problems try to solve them in various ways. In this way the pupil is taking a positive action in learning and should learn more readily and efficiently than by merely following instructions.

Dr. George Baird discussed the implementation of curricula. If one is to change the curricula in many schools in a limited span of time, this is clearly a job related closely to changing a business to a new product insofar as organization is concerned. Here Dr. Baird made several observations on what to do and what not to do. For example, teachers should not write materials nor should mathematical scholars. He also opposed giving away of copyrighted materials; such materials must be paid for, he says. Evaluations can and should be made by objective tests, is his claim.

Concerning the prospects raised by Dr. Baird's lecture, I find myself in conflict. It would seem preferable, to me, to have experiments carried on over a period of years, to decide on the workability of proposals for change. However if, or seems the case, the demands of society are going oscillate with a short period then long term experimentation would be fruitless, since the goals of education would change too much to permit leisurely experiments. I have not too much faith in our ability to define in detail, the objectives of mathematics education so that the definition will stand for many years.

The reasons for my skepticism are as follows:

1. There have been inadequate attempts to display the structure of mathematics. My "Chart of Elemental Mathematics" and "Organization of Mathematical Systems" are in this direction but are not adequately developed. If the structure is unknown how can a rational system be developed?

2. The concepts of mathematics have been divorced from the corresponding ones from real life. In a few cases I have begun spanning the gaps, - for example in cases of continuity, filters, approximation, function, neighborhood but this work has not penetrated the textbooks even on a collegiate level. On the other hand, increased use of models and applications are going on.

3. The improper emphasis on axiom systems and the failure to recognize the limited scope of mathematics indicates inadequate philosophies.

4. The failure to remove errors in notation, grammar, and logic from textbooks even at the collegiate level shows that the hurriedness has taken its toll. It seems to me that the system now in operation has not permitted time to weed out errors or we have not recognized the advisability of doing so.

5. The absence of any standards committee national or international for mathematical terminology means that language fuzziness will increase.

Concluding remarks.

This conference did the task for which it was designed. It showed that the middle school mathematics curriculum is a problem area for which helpful studies should be possible. The reforms suggested were not radical in general character and hence there might be a possibility of improving the quality of materials while reorganizing them into curriculum packages. The background problem to which J. Hlavaty repeatedly returned, is of finding in a more general setting, what the role of mathematics in education should be is important. Unfortunately this problem cannot be solved before work begins on the mathematical programs.

While there are many ways in which one could hope that the mathematical scholars and research workers would cooperate more fully with educators, it is none the less true that more interplay between scholars and educators is taking place now then there has been in the past.

All of us are much indebted to Professors Earl Beard and George Cunningham and to the National Science Foundation for enabling us to meet.

AREAS OF CONCERN BROUGHT OUT BY CONFERENCE DISCUSSIONS

The Envolving Philosophy of the Middle School and Its Implication for the Mathematics Curriculum

Several participants expressed the view that the middle school was beginning to develop a philosophy unique to the particular organization or suborganization of the middle school. There appears to be a trend in the middle schools to develop the curriculum around social problems and issues and that the curriculum must move toward overviews rather than beginning with basics. This type of curriculum organization virtually eliminated the traditional textbook approach and mathematics teachers have been slow to participate in its development.

The ensuing discussion brought out the following points:

> In a curriculum based on major issues of interest to the student and to society mathematics may disappear from the middle school curriculum unless mathematics teachers learn how to incorporate their subject matter into socially oriented topics.

> The idea that mathematics might virtually disappear from the curriculum was rediculous since mathematics is one of the tools of learning.

> Since mathematics, like reading, is a tool of

learning and should not be taught any when it pertains to some investigation of a social problem.

Mathematics is different from other subjects and must be taught separately. Thus a certain amount of classroom time should be devoted to mathematics every day.

The highly sequential nature of mathematics does not adapt readily to the casual and incidental learning characteristic of the above described curriculum.

It was suggested that the importance of the particular middle school curriculum is so great that the conference should recommend a study of the philosophy of the middle school rather than a study of subject matter which may be inappropriate for the schools.

Other thoughts that the alledged threat to the existance of mathematics in the middle school curriculum should be met in one of the following ways.

1. Influence or infiltrate the middle school movement to insure that its socially oriented core includes significant attention to the roles of mathematical reasoning.
2. Oppose the middle school movement.

3. Provide materials that will enable teachers to effectively use mathematics in the middle school.

Some participants thought it unreasonable to expect significant computational competence if mathematics is taught incidentally to an investigation of the Mediterranean Sea.

There was a difference of opinion as to whether mathematics must justify itself in terms of a particular middle school curriculum. Some expressed the view that the overview of the middle school curriculum must be justified in terms of the extent to which it facilitates the learning of reading, mathematics, and other fundamental subject matter.

No particular resolution of the apparently conflicting viewpoints was reached.

The Computation Skill or Lack of it Exhibited by Pupils.

It was noted that the public is concerned about an alleged lack of computation skill. It was alleged that standardized tests of computation have definitely shown a significant drop in this skill in recent years and that some of the public and some mathematicians attribute this drop to the introduction of modern mathematics into the school. The ensuing discussion brought out the following points to which all participants did not necessarily agree.

1. Test scores on applications of mathematics and on concepts seem to have improved.

2. Computation should not be taught or measured for its own sake but for its application to alter problems.

3. The public is demanding that school teach a better mastery of computation.

4. The computer and calculators have eliminated the necessity for computational skills.

5. Modern mathematics never intended to eliminate computation.

6. Teachers have neglected computations because they failed to understand the real goals of modern mathematics.

A Need For Establishing an Authoritative Statement of Philosophy and Goals of Mathematics Education.

This discussion seemed to have two interesting aspects. On the one hand, the view was expressed that there was an immediate need for the establishment of a set of goals and philosophy of American education. On the other hand, the view was stated that no committee should produce an authoritative statement of the goals and purposes of mathematics education.

A proposed justification for teaching mathematics listed five philosophical foundations.

1. Mathematics is an intellectual discipline, not a collection of algorithms.

2. The subject of mathematics is ideas, not notations.

3. Mathematics is an organized body of knowledge.

4. Mathematics give understanding and power in the real world.

5. Mathematics is a form of artistic expression.

A review of the accomplishments and failures of the school revolution in mathematics of the sixties was overdue.

It was not apparent that there would be any agreement as to how such a committee should be composed.

Interdisciplinary approach.

A discussion of the advantages and disadvantages of an interdisciplinary approach brought out the following points.

Disadvantages

1. Mathematics looses identity in an interdisciplinary approach
2. The interdisciplinary approach is not appropriate to the special nature of mathematics.
3. Mathematics requires a stable and established curriculum.
4. The interdisciplinary approach is not appropriate for all middle school organizations.
5. The interdisciplinary approach requires participation of other disciplines.
6. The interdisciplinary approach requires information about the present state of other disciplines.

Advantages

1. Relevancy and stability
2. Transfer of ideas and skills
3. The interdisciplinary approach relates to life and its problems
4. The interdisciplinary approach meets current development of middle school

5. The interdiciplinary approach identifies math needs for related disciplines
6. The interdisciplinary approach particularly appropriate for middle school approach
7. The interdisciplinary approach provides for the role of history of math, computers, calculators.

PARTICIPANT CONTRIBUTIONS

The following statements were submitted by participants during and following the conference. These statements represent the views of the authors noted and those who support these views.

The following statement was submitted by:
Julius H. Hlavaty

Concerning the determination of a committee to establish philosophical goals.

1. To explore the mathematical and mathematical education aspects of the problem the appropriate body to name and direct the committee would be the Conference Board of the Mathematical Sciences being as it is a federation of associations of mathematicians. teachers, statisticians, computer people, etc.

2. To place the problem in the necessary broad context of over-all general education and national problems and needs one or both of the following agencies would be appropriate:

a. A reactivated Educational Policies Commission of the National Education Association;

b. A commission appointed by the National Academy of Sciences.

In either case there should be representation of various disciplines (the sciences, including mathematics, the arts, the social sciences, etc.) and of various concerned groups (research people, teachers, administrators, the general public, etc.)

The following statement was submitted by:
Leslie P. Steffee

I am writing to you concerning the Conference on the Middle School Mathematics Curriculum held last week (July 16-20, 1973). The reason for writing is to express to you my sentiments about the conference and what we accomplished. Holding a conference is no easy task, from the inception of the idea to its culmination. So you are to be complimented for the splendid job you did (I should include also George Cunningham). The food was excellent, the hospitality warm, and Maine was beautiful. We also accomplished a great deal toward the identification and resolution of the so called "Middle School Problem." The resolution is particularly important, as is its rationale and recommendations for implementation. These three products of the conference are most important products insofar as positive action by the NSF is concerned. In short, I feel the conference was most timely and its products of great significance. Now, let me also express some sentiments of my view of the papers and surrounding discussions. Henry Van Engen's paper contained significant principles for those engaged in future curriculum development. However, I feel that Henry did not address himself specifically to mathematics curriculum In the Emerging Middle School, nor did he address himself to the critical issues involved in the training of mathematics teachers for the Emerging Middle School. Donovan Johnson's paper contained summary information (presented in a quite well organized fashion) concerning facets of a curriculum, delineating various approaches to curriculum development, but again he did not address himself specifically to the mathematics curriculum In the Emerging Middle School, nor did he address himself to the critical issues involved in the training of mathematics teachers for The Emerging Middle School. George Baird's paper contained information regarding an industrialist's approach to "educational research," but again he did not address himself specifically to the mathematics curriculum In the Emerging Middle School, nor did he address himself to the critical issues involved in the training of mathematics teachers for The Emerging Middle School. All of the major papers had something to say concerning mathematics education and were related in an indirect way to the problem at hand, but did not specifically deal with that problem. I can only speak about the discussion group I was in, but we did not deal directly with the problem at hand, but rather dealt with "mathematics for every citizen," "why teach mathematics," and an operational way of implementing the resolution adopted. Now, you might say "Where were you?" but the major papers rather set the tone for the discussions. These discussions dealt with problems again indirectly related to the problems of The Emerging Middle School (the first seems most relevant). But it was not until the General Session late Thursday afternoon that the conferees as a group in any way dealt with the problems of The Emerging Middle School. As a result of this condition, we have a weak conference report in terms of substantive identification of the problems associated with the Middle School. In retrospect, it ould have been good (in my opinion) to have had the opening address by an Emerging Middle School Advocate. But that is too late. In short, the major Paper presenters came with solutions to problems different than the resolution as well as other documents. As delightful as Preston Hammer's presentation was, he did not summarize the conference, but rather his paper distributed earlier.

My comments are not to be taken as critical of the conference, but are meant to be interpreted as a plea for the proposed work-study group to focus on the problem we only barely identified--the problem of the mathematics curriculum in the Emerging Middle School and the more severe problem of the preservice preparation of the Emerging Middle School teacher of mathematics and the research that needs to be done concerning those two central problems.

The following statement was submitted by:
Marguerite Brydegaard

MATHEMATICS IN THE MIDDLE GRADES -

SOME QUESTIONS AND COMMENTS FOLLOWING THE ORONO CONFERENCE

The British Primary School Movement is making a fine contribution through its development of programs for the early years of education. I spent considerable time visiting and participating in a number of schools including Sea Mill School where Max Beberman and his staff were located. Not unlike others who visited schools in England, I returned home with admiration and with inspiration to do better teaching. Classroom teachers were freed to teach all subjects - including mathematics. Teachers were apt in seeing mathematics in the total program and were able to identify specific important things in the curriculum that they were creating and building.

The National Middle School Movement in U.S.A. offers even greater opportunities for contributions than that of the British Primary School. The ideas that develop out of the Orono Conterence may point to new dimensions for study and action.

The following questions may merit continued consideration: Do we need to look more specifically at the total structure of the middle school in order to consider the mathematics curriculum for that level? Does the movement point to new curriculum ideas and new ways of teaching?

Are there new opportunities for real mathematicking if the teacher is ready to teach? Does the opportunity demand greater teacher understanding and broader interpretation of mathematics and of the nature of learning?

Do concepts of quantity and relationships among quantities permeate all subject matter? And, are these concepts likely to be very lightly touched upon by teachers and pupils who are unaware of the basic ideas and of procedures for investigating and evaluating mathematical ideas? For example, pupils are fascinated with ideas of gear ratios, pulleys, measurements, what the chances are that given things will happen, and so forth. Formal, book-type learning of mathematics often blocks the learning of exciting, real mathematics that is possible in a classroom. The middle school movement may open opportunities for new topics and new ways of teaching them.

Should the Middle School Movement lead each discipline to take a new look at itself and to identify its key concepts and significant behaviors that produce competence of learners? After their studies are completed, a gathering of several experts from each discipline should lead to some promising, productive results. For example, Guilford and Brittain each did separate and individual research to identify the attributes that are responsible for general creativeness in human behavior. Guilford's investigation was in the field of science (U. of Southern California) and Brittain's in the visual arts (Penn.State Univ.). The research in the two studies revealed that the attributes underlying creative thought as measured in their tests were essentially the same for science and for the art process. The two lists of attributes were the same; only their statements of the criteria were each in their individual wording.

What are specific procedures for utilizing classroom teacher leadership? A group of superior classroom teachers (they do exist), excellent mathematics educators (there are some), alert parents and perhaps a mathematician who has fine understanding of the learner and the learning process in addition to respect from the mathematics community would make a fine composition to study and report recommendations concerning mathematics in the middle school.

How can preservice and inservice teacher competence in interpreting and appreciating mathematics be facilitated?

Do hours of drilling upon the topics of number and operations deaden pupil appetite for study of mathematics? Is the great emphasis upon review that is not a re-view (a new view) but the same old stuff that had dulled pupil appreciation in the earlier years one of the greatest deterients of pupil interest in the middle school? If specific facts and skill are basic for pupils, they should be identified and new procedures for achieving mastery of them investigated. The analyses of pupil textbooks in terms of types of examples and the number of repetitions of examples for given types reveals the need for serious research concerning this issue.

The following statement was submitted by:
Jack Price

Prior to the conference I had our professional librarian conduct a computer search of the literature for material on mathematics and the middle school. She could locate only one general article pertaining to these descriptor pairs. However, more than 250 were found dealing with the middle school itself. This says that the mathematics community has had little input or output dealing with the middle school movement, I assumed that one purpose of this conference was to change that situation. Secondly, through visits to schools and through journal articles, it has become evident to me that the interdisciplinary approach in middle school curriculum is gaining headway. In many cases, this means formal mathematics instruction is being effectively removed from the curriculum. A way to compensate for or to remedy this loss, I felt, should have been an outcome of this conference.

But where are we? At the moment, we are standing outside the candy shop with our noses pressed against the glass. If the door is opened to us, I hope that we have the confidence to act constructively once inside. We know the philosophical bases for the middle school movement, and I believe that we have no choice but to accept it and work within it. If we compose a curriculum and impose it on the middle school, it will not be implemented. "They" have as much a vested interest in the success of "their" program as "we" do in the perpetuation of "ours."

What I have heard the last three days is content independent of organization--mathematics as we conceive it for everyone. The middle school reply is that students have the right to do some "post-holing" --depth rather than breadth--in some areas and some cafteria browsing in others. All students do not fit the same mold, and the true middle school attempts to provide a "personalized" education for each student. We will little serve the cause of mathematics or students by insisting upon one mathematics for all middle school students.

If there is a follow-up to this conference, and I assume there will be, I would recommend that the succeeding conference be more broadly based. It will need the input of the leaders of the middle school movement and the enthusiasm of the outstanding teachers as well as the mathematicians and mathematics educators assembled here. Perhaps students, too, should be invited. Working teams should be a cross section of the participants which in turn should be a cross section of the leadership in all aspects of mathematics and middle schools.

This should not be construed as a criticism of the conference, but there seemed to be a general lack of knowledge of the middle school movement among the participants. In addition, there was little idea how curriculum is developed and implemented at the local districts level. There is no lack of talent in school districts either to write curriculum or implement it thoroughly. After all, many of these people were students of the participants. Many of them have degrees in mathematics and/or mathematics education. Only the most dyed-in-the-wool chauvinists could truly believe that no good work can exist apart from the university. There is no lack of leadership at the local level. To say so shows a real lack of understanding and a supreme naivete!

There is a lack of funding brought on in part by the tax exempt status of certain foundations which sell materials to publishers and the unwillingness of NSF to provide funds to local districts. The plain fact is that much good curriculum has been written by teachers in districts under the direction of highly competent specialists. The curriculum development is limited only by the resources of the district. There has been a tendency over the past few days to write off teachers as developers of curriculum--and it should not be done.

The following statement was submitted by:
Joseph Scandura

"While I am not opposed to a study group to determine what ought to be done in the middle school, I do not know whether this would serve a useful purpose, especially if extreme care is not taken to ensure that the selected group represents all relevant positions in approximately equal proportion.

More important, I feel that the money might better be spent on actual projects aimed at improving middle school mathematics (as well as mathematics education generally). In our working group it was agreed that materials and instruction development should be based on more clearly defined and well developed rationales. Among other things more attention should be given to defining objectives, particularly the higher level processes characteristic of mathematical thinking. I personally also believe that such projects should be directed at immediate and felt needs in the schools.

Our group also recommended that relevant research be supported. The following problems were mentioned specifically: methods of testing (particularly of higher processes), problem solving, mathematics learning and instruction, and the development of mathematical applications. To this I would add research on the relationships between research and materials and instruction development and teacher education in mathematics, and specifically the development of new and better methods for amaterials and instruction development and teacher education."

The following statement was submitted by:
Donald Dessart

In reviewing the resolution and the rationale for the resolution again, I am even more firmly convinced of its reasonableness and necessity. Without sounding too much like a broken record, I sincerely hope that item (2) of the rationale (concerning interdisciplinary work with the sciences and social sciences) is given very thorough consideration by a study group composed of educators from mathematics as well as disciplines other than mathematics. The whole prospect sounds exciting!

The following statement was submitted by:
Thomas Romberg

Although I voted for the resolution adopted by this Conference and the three reasons put forward in its support, I am concerned that neither the resolution nor the reasons refer to the principal problem of instruction in our schools.

The problem I am referring to is particularly evident in Middle Schools and is one which modern mathematics programs have inadvertently, but significantly contributed to. The problem is the dehumanization of contempory classrooms. That may, if not most, schools deliberately foster a nightmarish learning enviroment that is joyless and repressive has been well documented. Modern mathematics materials have contributed to this situation by stressing rigor instead of relevance, by failing to adequately consider child learning and development, and by providing materials which in the hands of typical teachers could not be well used.

If the work-study group approaches its work from a perspective of humanizing mathematics, then my fears, are groundless. If, however, the group only considers new content or teacher training, then I fear it will be a fruitless effort.

Conference Conclusions

After much discussion and confusion it was apparent to the participants that there were problems in the middle school mathematics curriculum, and that there was no unanimity as to the solutions. There was a feeling that there problems were solvable and were deserving of solution and so the participants agreed to the following resolution.

> The members attending the Orono Conference on the Middle School Mathematics Curriculum at the University of Maine, July 16-20, 1973, recommend that the National Science Foundation finance a work-study group to study the mathematics curriculum (K-12) with special emphasis on the middle school problems. To study is to include emerging curriculum patterns and materials and all associated problems. The results of this study are to be widely dissemin-

ated prior to September, 1975.

A few of the many reasons why the participants felt the above resolution compelling were summarized by J. Kilpatrick and H. O. Pollak and are:

(1) A representative national group needs to consider the curriculum of the middle school from the point of view of putting first those concepts of mathematics most important to <u>all</u> students. This may, for example, lead to a radically new emphasis for statistics, proba-

bility, computing, and other subjects not now typically part of the middle school program. Such a study is particularly appropriate in view of the emphasis of most recent curriculum work on college-capable students, and the fact that grades 8 and 9 are likely to be the last time all students take mathematics together.

(2) Mathematics teachers in the middle school are increasingly being asked to contribute to interdisciplinary work in the sciences and social sciences. The problems of curriculum, pedagogy, educational philosophy, and teacher training which are raised by this exciting new trend need to be examined. First of all, it is far from clear what aspects of the mathematics curriculum could, or should, be handled in this fashion. Once this has been studied, questions of preparing teachers for interdisciplinary work, and providing materials that would be most useful to them, can then be attacked.

(3) Middle school mathematics has a number of fundamental aspects including, for example, quantitative reasoning, spatial perception, mathematical technique, structure, and model building. Different ones of these are stressed in different current curriculum efforts. Teachers, however,

want help in putting together the output of these projects into a balanced mathematics program for the middle school. In particular, no proper subset of these aspects should be allowed to eliminate its complement from the mathematics curriculum.

Although agreeing with the resolution, a number of participants would have modified it to reflect areas of special interest or concern. They would have liked to see mention of specific problems like teacher training, national uniformity, interdisciplinary courses, etc., in place of "all associated problems." In general, it was felt that what was needed was a broad statement of need and direction so as to permit any investigative group the widest possible latitude.

After the work of the conference was completed, the participants indicated their appreciation to all concerned with the organization of the conference with an appropriate statement.